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Design of a Control Chart for Gamma Distributed Variables Under the Indeterminate Environment

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ABSTRACT In this paper, the design of the control chart when the variable of interest follows the gamma distribution under the neutrosophic statistical interval method (NSIM) is proposed. The average run length, probability in-control, and probability of out-of-control will be derived using the NSIM. The neutrosophic control chart coefficient will be determined by the algorithm under the NSIM. The neutrosophic average run length for various shifts and specified parameters will be determined. The efficiency of the proposed control chart is discussed using the simulation study and a real example.

INDEX TERMS Neutrosophic statistics, classical statistics, fuzzy logic, neutrosophic logic, neutrosophic average run length.

I. INTRODUCTION

High quality is the main aim of well-reputed industries or service companies in the modern world. The high-quality target can be only achieved by minimizing the non-conforming or the defective items through a proper monitoring of the manufacturing process. The proper monitoring of the process can be done using the control charts. The control charts analyzed the process, help the industrial engineers, and provide information to management in the variety of ways such as when the corrective action should be taken, unexpected shift in the process and pinpointing the causes of variation in the process. Several authors designed a control chart for the various reasons, for example, [1] discussed the application of the control chart in medicine and epidemiology. Reference [2] applied the control chart in dairy form. Detailed applications for healthcare issues can be seen in [3].

The Shewhart control charts are designed under the assumption that quality of interest follows the normal distribution. But, the several manufacturing processes such as chemical, semiconductor, cutting tool wear and accelerated test the distribution of variable of interest is skewed, see [4] and [5]. According to [6], the use of the Shewhart control chart when data is not collected in the subgroup or when the variable of interest follows the skewed distribution may mislead the industrial engineers in interpreting the state of the control chart. The gamma distribution is an appropriate model for the skewed data and used to model the time between events. Several authors worked on the control chart charts for the gamma distribution, for example, [7] designed the R chart for the gamma distribution. Reference [8] designed control chart when the quality of interest follows the gamma distribution. Reference [9] worked on the control chart when the time between follows the gamma distribution. Reference [10] proposed control for the gamma distribution using the belief information. Reference [11] studied the control chart for this distribution using a narrow confidence interval method. References [12] and [13] designed a chart for the gamma distribution under resampling approaches.

The fuzzy logic is applied in the designing of control charts when the proportion of defective parameter is not determined value. According to [14] ''The use of a fuzzy approach in the design of control charts has allowed to improve the performance of traditional charts, as well as to be possible of a simple approach for the design of control charts for linguistic variables with multinomial distribution for both, the univariate case as for the multivariate case''. Due to the importance of the fuzzy- based control charts in the industry, several authors designed control chart based on fuzzy logic including for example, [15] and [16] introduced the fuzzy-based algorithm to improve the process. Reference [17] worked on fuzzy control chart for skewed distribution. Reference [18] discussed the cost model for the Weibull distribution under the fuzzy approach. Reference [19] designed control chart for the fuzzy random numbers. Reference [20] worked

on the fuzzy X-bar chart using non-normal distribution. Reference [21] control chart for multinomial control chart using fuzzy logic.

Reference [22] developed the generalized form of fuzzy logic is called the neutrosophic logic. The neutrosophic logic becomes the traditional fuzzy logic when there are indeterminate observations in the sample or in the population. Reference [23] developed the generalized class of the classical statistics using the neutrosophic logic is called the neutrosophic statistics (NS). The NS reduced to classical statistics when all the data is clear and determined. References [24] and [25] used the NS to study rock measuring issues. References [26]–[28] introduced the NS in the area of acceptance sampling plans. Recently, [29] introduced the attribute control chart based on the NS. Reference [30] proposed the variance control chart using the neutrosophic statistical interval method (NISM). More details can be seen in [30].

The existing control charts for the gamma distribution under the classical statistics cannot be applied when the observations are fuzzy, indeterminate and unclear. Therefore, our aim is to design a control chart to monitor the gamma-distributed quality of interest under the uncertainty environment. By exploring the literature and the best of our knowledge, there is no work on the control chart for the gamma distribution under the NISM. In this article, the design of the control chart when the variable of interest follows the gamma distribution under the NSIM is proposed. The average run length, probability in-control and, probability of out-of-control will be derived using NSIM. The neutrosophic control chart coefficient will be determined the algorithm under the NSIM. The neutrosophic average run length (NARL) for various shifts and specified parameters will be determined. The efficiency of the proposed control chart is discussed using the simulation study and a real example.

II. NEUTROSOPHIC GAMMA DISTRIBUTION

Suppose that the neutrosophic life/failure time $T_N \in [T_L, T_U]$, where T_L and T_U present the lower and upper failure of indeterminacy interval of an item follows the neutrosophic gamma distribution with neutrosophic shape parameter $a_N \in [a_L, a_U]$ and neutrosophic scale parameter $b_N \in [b_L, b_U]$. The neutrosophic probability density function (npdf) of the neutrosophic gamma distribution (NGD) is given by

$$f(t_N) = \frac{b_N^{a_N}}{\Gamma(a_N)} t_N^{a_N-1} \mathrm{e}^{-b_N t_N}; \quad t_N, a_N, b_N > 0; \quad a_N \in [a_L, a_U], \quad b_N \in [b_L, b_U] \quad (1)$$

where $\Gamma(x)$ presents the neutrosophic gamma function, see [28]. The corresponding neutrosophic cumulative distribution function (ncdf) of NGD is given by

$$P(T_N \le t_N) = 1 - \sum_{\mathrm{j}=1}^{a_N-1} \frac{\mathrm{e}^{-\frac{t_N}{b_N}} (t_N/b_N)^{\mathrm{j}}}{\mathrm{j}!}; \quad T_N \in [T_L, T_U], \quad a_N \in [a_L, a_U], \ b_N \in [b_L, b_U] \quad (2)$$

Note here that the NGD is the generalization of traditional gamma distribution under the classical statistics. The NGD becomes the traditional gamma distribution when all the observations or the parameters are determined values. The neutrosophic mean and neutrosophic variance of NGD are given by

$$\mu_N = \frac{a_N}{b_N}; \quad a_N \in [a_L, a_U], \ b_N \in [b_L, b_U] \quad (3)$$

and

$$\sigma_N^2 = \frac{a_N}{b_N^2}; \quad a_N \in [a_L, a_U], \ b_N \in [b_L, b_U] \quad (4)$$

By following [31], the NGD can be transformed to an approximately neutrosophic normal distribution when $T_N^* = T_N^{1/3}; T_N \in [T_L, T_U]$. More details about the neutrosophic distributions can be seen in [23] and [26]. The neutrosophic mean and neutrosophic variance of $T_N^* \in [T_L^*, T_U^*]$ are given by

$$\mu_{T_N^*} = \frac{b_N^{1/3}\Gamma(a_N + 1/3)}{\Gamma(a_N)}; \quad a_N \in [a_L, a_U], \ b_N \in [b_L, b_U] \quad (5)$$

and

$$\sigma_{T_N^*} = \frac{b_N^{2/3}\Gamma(a_N + 2/3)}{\Gamma(a_N)} - \left(\frac{b_N^{1/3}\Gamma(a_N + 1/3)}{\Gamma(a_N)}\right)^2; \quad a_N \in [a_L, a_U], \quad b_N \in [b_L, b_U] \quad (6)$$

III. DESIGN OF THE PROPOSED CONTROL CHART

As mentioned by [31], the transformed vaiable $T_N^* = T_N^{1/3}; T_N^* \in [T_L, T_U]$ have the symmetry property of the neutrosophic normal distribution. The ncdf is valid for integer values of a_N. We propose the following control chart under the NISM when the quality of interest follows the NGD.

1. Compute $T_N^* = T_N^{1/3}$ for the randomly selected item from the production process.
2. Plot T_N^* and declare the process out-of-control if $T_N^* \ge UCL_N$ or $T_N^* \le LCL_N$; where $\mathrm{LCL_N} \in [LCL_L, LCL_U]$ and $\mathrm{UCL_N} \in [UCL_L, UCL_U]$ are neutrosophic lower control limit and neutrosophic upper control limit, respectively.

The proposed control chart under the NISM is the extension of the control chart proposed by [8] under the classical statistics. The proposed control chart becomes [8] chart when no uncertain observations or parameters exist. Suppose now that the process is in in-control state at neutrosophic scale parameter $\mathrm{b_{0N}} \in [b_{0L}, b_{0U}]$. The neutrosophic control limits for the in-control process are given by

$$\mathrm{LCL_N} = \mu_{T_N^*} - \mathrm{k_N}\sigma_{T_N^*} = \frac{\mathrm{b_{0N}^{1/3}}\Gamma\left(\mathrm{a}_N + \frac{1}{3}\right)}{\Gamma(\mathrm{a}_N)} - \mathrm{k_N}\sqrt{\frac{\mathrm{b_{0N}^{2/3}}\Gamma(\mathrm{a}_N + 2/3)}{\Gamma(\mathrm{a}_N)} - \mu_{T_N^*}^2} \quad (7)$$

$$\mathrm{UCL_N} = \mu_{T_N^*} + \mathrm{k_N}\sigma_{T_N^*} = \frac{\mathrm{b_{0N}^{1/3}}\Gamma(\mathrm{a}_N + 1/3)}{\Gamma(\mathrm{a}_N)} + \mathrm{k_N}\sqrt{\frac{\mathrm{b_{0N}^{2/3}}\Gamma(\mathrm{a}_N + 2/3)}{\Gamma(\mathrm{a}_N)} - \mu_{T_N^*}^2} \tag{8}$$

where $\mathrm{k_N} \in [k_L, k_U]$ is the neutrosophic control limit coefficient.

Let, we define

$$\mathrm{LL_N} = \left[\frac{\Gamma(\mathrm{a}_N+1/3)}{\Gamma(\mathrm{a}_N)} - \mathrm{k_N}\sqrt{\frac{\Gamma(\mathrm{a}_N+2/3)}{\Gamma(\mathrm{a}_N)} - \left(\frac{\Gamma(\mathrm{a}_N + 1/3)}{\Gamma(\mathrm{a}_N)}\right)^2}\right]$$

$$\mathrm{UL_N} = \left[\frac{\Gamma(\mathrm{a}_N+1/3)}{\Gamma(\mathrm{a}_N)} + \mathrm{k_N}\sqrt{\frac{\Gamma(\mathrm{a}_N+2/3)}{\Gamma(\mathrm{a}_N)} - \left(\frac{\Gamma(\mathrm{a}_N + 1/3)}{\Gamma(\mathrm{a}_N)}\right)^2}\right]$$

Therefore, the neutrosophic control limits can be written as follows

$$\mathrm{LCL_N} = \mathrm{b_{0N}^{1/3}LL_N} \tag{9}$$

$$\mathrm{UCL_N} = \mathrm{b_{0N}^{1/3}UL_N} \tag{10}$$

Note here that the when the process is shifted only the neutrosophic scale parameters of the proposed chart will be changed. In practice, the neutrosophic shape parameter is usually known and or fixed accordingly. The probability of in-control when the process is at $\mathrm{b_{0N}} \in [b_{0L}, b_{0U}]$ under the NISM is derived as follows

$$\mathrm{P_{out,N}^0} = \mathrm{P}\left(T_N^* < \mathrm{LCL_N}|\mathrm{b}_N = \mathrm{b_{0N}}\right) + \mathrm{P}\left(T_N^* > UCL_N|\mathrm{b}_N = \mathrm{b_{0N}}\right) \tag{11}$$

or

$$\mathrm{P_{out,N}^0} = 1 - \sum\nolimits_{\mathrm{j}=1}^{\mathrm{a}_N-1} \frac{\mathrm{e}^{-LL_N^3}(\mathrm{LL_N}^3)^{\mathrm{j}}}{\mathrm{j}!} + \sum\nolimits_{\mathrm{j}=1}^{\mathrm{a}-1} \frac{\mathrm{e}^{-UL_N^3}(UL_N^3)^{\mathrm{j}}}{\mathrm{j}!} \tag{12}$$

We will measure the efficiency of the proposed control chart under the neutrosophic average run length (NARL) which shows on the average when the process is out-of-control is defined by

$$\mathrm{ARL_{0N}} = \frac{1}{\mathrm{P_{out,N}^0}}; \quad \mathrm{ARL_{0N}} \in [ARL_{0L}, ARL_{0U}] \tag{13}$$

Suppose that due to some special causes of variations, the process has shifted from the targeted $\mathrm{b_{0N}} \in [b_{0L}, b_{0U}]$ to $\mathrm{b_{1N}} = c\mathrm{b_{0N}}$; $\mathrm{b_{1N}} \in [b_{1L}, b_{1U}]$, where the constant c shows the shift in the process. The probability of in-control when the process is at $\mathrm{b_{1N}} \in [b_{1L}, b_{1U}]$ under the NISM is derived as follows

$$\mathrm{P_{out,N}^1} = \mathrm{P}\left(T_N^* < \mathrm{LCL_N}|\mathrm{b}_{1N} = c\mathrm{b_{0N}}\right) + \mathrm{P}\left(T_N^* > UCL_N|\mathrm{b}_{1N} = c\mathrm{b_{0N}}\right) \tag{14}$$

TABLE 1. The values of NARL when $\mathrm{ARL_{0N}}$ = 200, 300, 370.

k_N	[2.75,2.78]	[2.86,2.90]	[2.92,2.96]
a_N	[3,5]	[3,5]	[3,5]
b_N	[1.9,2.1]	[1.9,2.1]	[1.9,2.1]
c	NARL		
4	[1.82,1.33]	[1.92,1.37]	[1.984,1.405]
3	[2.80,1.88]	[3.05,1.99]	[3.197,2.071]
2.8	[3.21,2.12]	[3.53,2.27]	[3.712,2.367]
2.5	[4.13,2.68]	[4.63,2.92]	[4.916,3.078]
2.25	[5.46,3.51]	[6.23,3.90]	[6.673,4.152]
2	[7.85,5.08]	[9.17,5.79]	[9.943,6.241]
1.9	[9.37,6.12]	[11.08,7.05]	[12.079,7.657]
1.8	[11.45,7.58]	[13.71,8.85]	[15.048,9.683]
1.7	[14.38,9.71]	[17.47,11.51]	[19.317,12.699]
1.6	[18.65,12.95]	[23.04,15.62]	[25.691,17.398]
1.5	[25.14,18.13]	[31.65,22.31]	[35.642,25.132]
1.4	[35.42,26.92]	[45.63,33.91]	[51.977,38.7]
1.3	[52.46,42.78]	[69.48,55.43]	[80.272,64.263]
1.2	[81.69,72.96]	[111.95,97.87]	[131.618,115.703]
1.1	[130.88,129.32]	[187.17,181.02]	[224.98,219.282]
1	[200.02,206.71]	[300.03,301.98]	[370.011,375.064]
0.8	[233.69,168.52]	[365.21,247.09]	[461.386,307.94]
0.75	[205.77,132.67]	[320.63,193.16]	[404.591,239.851]
0.7	[174.29,101.44]	[270.55,146.69]	[340.814,181.489]
0.6	[116.07,55.81]	[178.81,79.51]	[224.456,97.608]
0.5	[71.43,28.42]	[109.06,39.69]	[136.345,48.22]
0.4	[40.02,13.22]	[60.32,17.94]	[74.964,21.464]
0.3	[19.59,5.55]	[28.91,7.20]	[35.581,8.41]
0.25	[12.75,3.48]	[18.51,4.36]	[22.609,5.004]
0.15	[4.35,1.41]	[5.94,1.58]	[7.048,1.709]
0.1	[2.22,1.06]	[2.83,1.102]	[3.246,1.132]
0.05	[1.148,1.00]	[1.262,1.00]	[1.343,1.001]

or

$$\mathrm{P_{out,N}^1} = 1 - \sum\nolimits_{\mathrm{j}=1}^{\mathrm{a}_N-1} \frac{\mathrm{e}^{\frac{-LL_N^3}{c}}\left(\frac{\mathrm{LL_N}^3}{c}\right)^{\mathrm{j}}}{\mathrm{j}!} + \sum\nolimits_{\mathrm{j}=1}^{\mathrm{a}-1} \frac{\mathrm{e}^{\frac{-UL_N^3}{c}}\left(\frac{UL_N^3}{c}\right)^{\mathrm{j}}}{\mathrm{j}!} \tag{15}$$

The NARL for the shifted process is defined as

$$\mathrm{ARL_{1N}} = \frac{1}{\mathrm{P_{out,N}^1}}; \quad \mathrm{ARL_{1N}} \in [ARL_{1L}, ARL_{1U}] \tag{16}$$

Suppose that r_{0N} be the specified value of $\mathrm{ARL_{0N}}$. The values of $\mathrm{ARL_{1N}}$ for various shift c, a_N and b_N are displayed in Tables 1-2. From Tables 1-2, we note that for the fixed values of $a_N \in [3, 5]$, $b_N \in [1.9, 2.1]$ and c, the indeterminacy interval range in $\mathrm{ARL_{1N}}$ increases as $\mathrm{ARL_{0N}}$ increases. We also note that indeterminacy interval range in $\mathrm{ARL_{1N}}$ increases when $a_N \in [3, 5]$; $b_N \in [1.9, 2.1]$ change to $a_N \in [5, 10]$; $b_N \in [1.45, 1.55]$.

The following neutrosophic algorithm is used to find $\mathrm{k_N} \in [k_L, k_U]$ and $\mathrm{ARL_{1N}} \in [ARL_{1L}, ARL_{1U}]$.

1. Specify the values of $a_N \in [a_L, a_U]$, $b_N \in [b_L, b_U]$ and c.

TABLE 2. The values of NARL when ARL_{0N} = 200, 300, 370].

k_N	[2.77,2.79]	[2.89,2.92]	[2.96,2.98]
a_N	[5,10]	[5,10]	[5,10]
b_N	[1.45,1.55]	[1.45,1.55]	[1.45,1.55]
c	NARL		
4	[1.33,1.04]	[1.37,1.05]	[1.40,1.05]
3	[1.87,1.21]	[1.99,1.24]	[2.06,1.26]
2.8	[2.10,1.29]	[2.27,1.34]	[2.36,1.37]
2.5	[2.66,1.52]	[2.92,1.60]	[3.06,1.64]
2.25	[3.48,1.871]	[3.89,2.01]	[4.13,2.08]
2	[5.02,2.57]	[5.77,2.84]	[6.21,2.99]
1.9	[6.04,3.06]	[7.03,3.43]	[7.61,3.64]
1.8	[7.48,3.78]	[8.83,4.30]	[9.62,4.59]
1.7	[9.57,4.86]	[11.48,5.63]	[12.62,6.06]
1.6	[12.74,6.59]	[15.57,7.79]	[17.28,8.49]
1.5	[17.81,9.55]	[22.23,11.57]	[24.94,12.75]
1.4	[26.39,15.03]	[33.78,18.74]	[38.38,20.94]
1.3	[41.83,26.22]	[55.18,33.81]	[63.67,38.42]
1.2	[71.13,51.57]	[97.38,69.31]	[114.49,80.41]
1.1	[125.6,111.45]	[179.98,157.93]	[216.67,188.06]
1	[200.04,203.21]	[300.02,302.98]	[370.01,370.18]
0.8	[163.05,92.11]	[245.46,132.99]	[303.72,160.07]
0.75	[128.44,61.92]	[191.91,88.21]	[236.61,105.49]
0.7	[98.27,40.97]	[145.75,57.52]	[179.08,68.31]
0.6	[54.14,17.37]	[79.03,23.54]	[96.35,27.50]
0.5	[27.61,7.18]	[39.46,9.302]	[47.63,10.62]
0.4	[12.88,3.04]	[17.84,3.69]	[21.22,4.09]
0.3	[5.43,1.48]	[7.17,1.64]	[8.32,1.75]
0.25	[3.41,1.16]	[4.34,1.23]	[4.96,1.27]
0.15	[1.40,1.00]	[1.58,1.00]	[1.70,1.00]
0.1	[1.059,1.00]	[1.10,1.00]	[1.13,1.00]
0.05	[1.00,1.00]	[1.00,1.00]	[1.00,1.00]

2. Determine $k_N \in [k_L, k_U]$ such that $ARL_{0N} \geq r_{0N}$.
3. Several combinations of $k_N \in [k_L, k_U]$ will exist where $ARL_{0N} \geq r_{0N}$.
4. Choose that values of $k_N \in [k_L, k_U]$ where ARL_{0N} is r_{0N} or very close to it.
5. Use the selected $k_N \in [k_L, k_U]$ to find $ARL_{1N} \in [ARL_{1L}, ARL_{1U}]$ at various values of c.

IV. ADVANTAGES OF THE PROPOSED CONTROL CHART

The efficiency of the proposed control chart under NISM with the control chart under the classical statistics will be discussed in this section. For the fair comparison, we will fix the same values of specified parameters for both control charts.

A. THEORETICAL COMPARISON

In this section, we show the efficiency of the proposed control chart under NISM over the existing control chart proposed by [8] under the classical statistics. Both control chart will be compared in NARL. The control chart having the smaller values of NARL is known as the most efficient chart. The smaller the NARL, quicker the indication about the shift in the process. Also, as pointed out by [24] that a method which has provided the interval range rather than the determined value of the parameters is known as most effective and adequate under the uncertainty environment. The values of $ARL_{1N} \in [ARL_{1L}, ARL_{1U}]$ of the proposed control chart and the existing control chart when $a_N \in [3, 5]$, $b_N \in [1.9, 2.1]$ and $ARL_{0N} \in [370, 370]$ are shown in Table 3.

TABLE 3. Comparison of proposed chart and existing chart.

c	ARL_{1N} of the proposed chart	ARLs for existing chart	c	ARL_{1N} of the proposed chart	ARLs for existing chart
4	[1.98,1.41]	1.99	1.2	[131.62,115.7]	134.32
3	[3.2,2.07]	3.21	1.1	[224.98,219.28]	230.25
2.8	[3.71,2.37]	3.73	1	[370.01,375.06]	379.93
2.5	[4.92,3.08]	4.95	0.8	[461.39,307.94]	475.27
2.25	[6.67,4.15]	6.73	0.75	[404.59,239.85]	416.71
2	[9.94,6.24]	10.04	0.7	[340.81,181.49]	350.95
1.9	[12.08,7.66]	12.21	0.6	[224.46,97.61]	231.03
1.8	[15.05,9.68]	15.22	0.5	[136.35,48.22]	140.27
1.7	[19.32,12.7]	19.56	0.4	[74.96,21.46]	77.06
1.6	[25.69,17.4]	26.04	0.3	[35.58,8.41]	36.53
1.5	[35.64,25.13]	36.17	0.25	[22.61,5]	23.19
1.4	[51.98,38.7]	52.83	0.15	[7.05,1.71]	7.20
1.3	[80.27,64.26]	81.74	0.1	[3.25,1.13]	3.30

From Table 3, it can be observed that the proposed control chart has smaller values of NARL as compared to the chart proposed by [8] at all levels of c. For an example, when $c = 0.7$, the indeterminacy interval of NARL is $ARL_{1N} \in [340.81, 181.49]$ while the existing control chart has a determined value of ARL is 350.95. Under the indeterminate environment, the proposed control chart indicates that the process will be shifted between 181st sample and 340th sample. On the other hand, the existing chart indicates that the process will be shifted at the 350th sample. By comparing both control chart, it is concluded that the proposed control chart not only provides the quick indication about the shift in the process but also provide the indeterminacy interval which is required under the uncertainty environment. Therefore, the theory of the proposed control chart matches with [24].

B. THEORETICAL COMPARISON

Now, we will compare the proposed control chart under the NISM with the existing control chart using the simulated data. The data is generated from the NGD when $a_N \in [3, 5]$, $b_N \in [1.9, 2.1]$ and $ARL_{0N} \in [370, 370]$. First 20, observations are generated from the NGD when the process is in-control state and next 30 observations are generated from the NGD at $c = 1.5$. At these specified parameters, the tabulated $ARL_{1N} \in [35.642, 25.132]$ which means, the first out-of-control can be expected between 25th and 35th samples. We determined statistic $T_N^* = T_N^{1/3}$; $T_N^* \in [T_L, T_U]$ and plot it on control chart in Figure 1. From Figure 1, it can be seen that the proposed control chart under NISM detects shift at the 25th sample. We also plotted the T_N^* under the classical statistics on the control chart in Figure 2. From Figure 2, we note

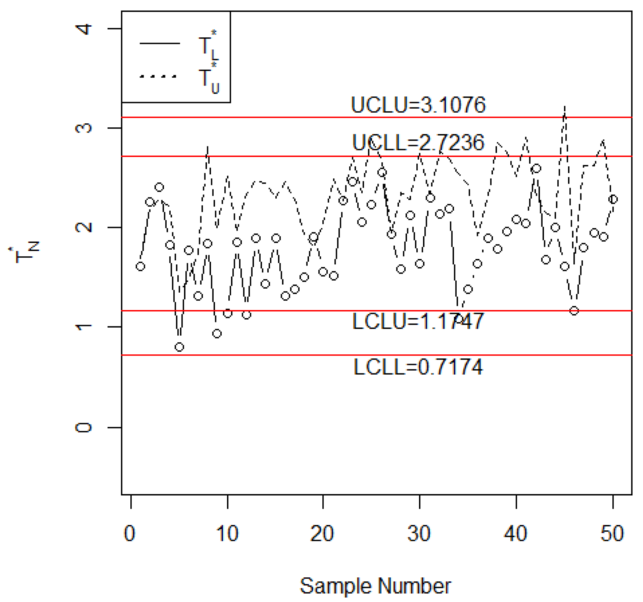

FIGURE 1. The proposed control chart for simulated data.

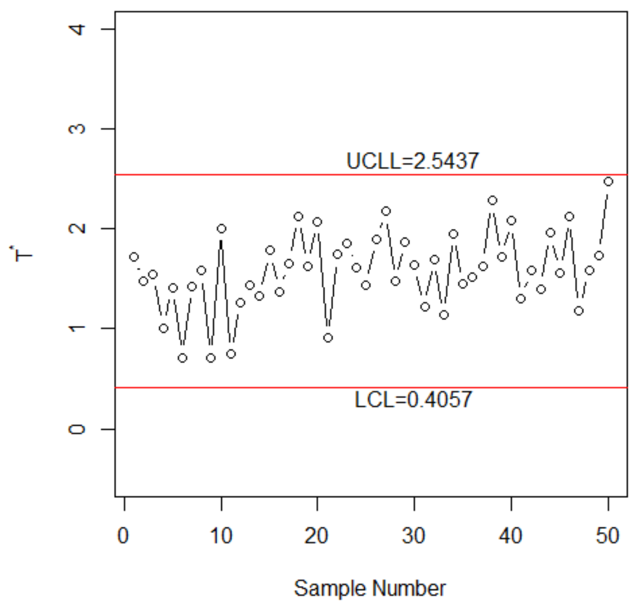

FIGURE 2. The existing control chart for the simulated data.

that the existing control chart does not provide any signal about the shift in the process. By comparing Figure 1 with Figure 2, we conclude that the proposed control chart is more efficient in detecting a shift in the process.

V. CASE STUDY

In this section, we will discuss the application of the proposed control chart under NISM in the healthcare department. The large hospital in Saudi Arabia wants to apply the proposed control chart for the monitoring of the urinary tract infections (UTIs). The hospital wants to monitor the increase or decrease infection rate in those patients who had acquired UTIs during their stay in the hospital. The microbes which are too small are the main reason for UTIs infection. As pointed out by [32] that the uncertainty exists while collecting the UTIs data based on the clinical diagnosis. Therefore, under the uncertainty environment, it is not possible that all the UTIs data is determined. So, for the UTIs data having some uncertain observations, the existing control chart using under the classical statistics cannot be applied for the monitoring of infection rate adequately and effectively. By following, [6], the data is collected in the number of days which follows the NGD with $a_N \in [5, 10], b_N \in [1.45, 1.55]$. The data along with the statistic $T_N^* = T_N^{1/3}; T_N^* \in [T_L, T_U]$ is shown in Table 4.

TABLE 4. The neutrosophic ATIs data.

Sr#	T_N	T_N^*	Sr#	T_N	T_N^*
1	[13.13,13.56]	[2.35,2.38]	26	[6.53,12.47]	[1.87,2.31]
2	[3.57,15.55]	[1.52,2.49]	27	[6.85,12.13]	[1.89,2.29]
3	[4.31,16.50]	[1.62,2.54]	28	[8.08,22.69]	[2.00,2.83]
4	[2.76,25.53]	[1.40,2.94]	29	[11.61,17.14]	[2.26,2.57]
5	[7.75,15.38]	[1.97,2.48]	30	[3.98,17.16]	[1.58,2.57]
6	[11.45,13.18]	[2.25,2.36]	31	[6.81,17.25]	[1.89,2.58]
7	[9.20,15.18]	[2.09,2.47]	32	[4.42,12.53]	[1.64,2.32]
8	[5.51,9.77]	[1.76,2.13]	33	[6.53,13.96]	[1.86,2.40]
9	[8.18,13.07]	[2.01,2.35]	34	[8.73,9.30]	[2.05,2.10]
10	[7.07,19.91]	[1.91,2.71]	35	[5.37,9.43]	[1.75,2.11]
11	[7.35,14.89]	[1.94,2.46]	36	[8.44,6.35]	[2.03,1.85]
12	[5.62,11.09]	[1.77,2.23]	37	[11.79,17.01]	[2.27,2.57]
13	[8.38,16.72]	[2.03,2.55]	38	[5.33,14.90]	[1.74,2.46]
14	[9.49,10.06]	[2.11,2.15]	39	[4.20,21.20]	[1.61,2.76]
15	[4.90,23.67]	[1.69,2.87]	40	[5.74,11.95]	[1.79,2.28]
16	[4.45,14.68]	[1.64,2.44]	41	[5.24,11.09]	[1.73,2.23]
17	[7.11,16.44]	[1.92,2.54]	42	[5.10,10.10]	[1.72,2.16]
18	[9.37,15.95]	[2.10,2.51]	43	[9.11,24.54]	[2.08,2.90]
19	[12.00,16.38]	[2.28,2.53]	44	[8.39,10.21]	[2.03,2.16]
20	[7.41,16.62]	[1.95,2.55]	45	[5.33,18.03]	[1.74,2.62]
21	[10.64,15.15]	[2.19,2.47]	46	[7.9,11.43]	[1.99,2.25]
22	[6.63,11.21]	[1.87,2.23]	47	[3.62,13.00]	[1.53,2.35]
23	[2.87,14.27]	[1.42,2.42]	48	[5.01,13.62]	[1.71,2.38]
24	[6.87,10.37]	[1.90,2.18]	49	[4.09,12.88]	[1.60,2.34]
25	[6.16,18.85]	[1.83,2.66]	50	[9.38,17.45]	[2.10,2.59]

Let $ARL_{0N} \in [370, 370], a_N \in [5, 10], b_{0N} \in [1.45, 1.55]$ and $k_N \in [2.96, 2.981]$. The neutrosophic control limits using this information is computed as follows

$$LCL_N = \frac{b_{0N}^{1/3}\Gamma\left(a_N + \frac{1}{3}\right)}{\Gamma(a_N)} - k_N\sqrt{\frac{b_{0N}^{2/3}\Gamma\left(a_N + \frac{2}{3}\right)}{\Gamma(a_N)} - \mu_{T_N^*}^2};$$

$$LCL_N \in [1.0489, 2.7361]$$

$$UCL_N = \frac{b_{0N}^{1/3}\Gamma(a_N + 1/3)}{\Gamma(a_N)} + k_N\sqrt{\frac{b_{0N}^{2/3}\Gamma(a_N + 2/3)}{\Gamma(a_N)} - \mu_{T_N^*}^2};$$

$$UCL_N \in [1.6863, 2.7361]$$

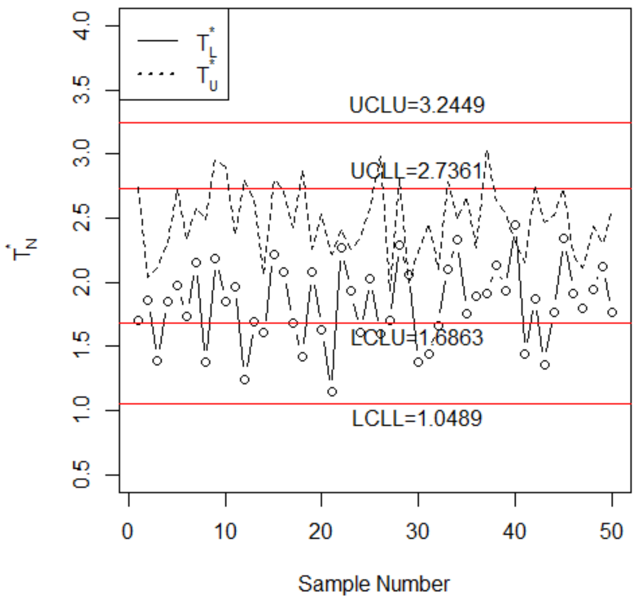

FIGURE 3. The proposed control chart for UTIs data.

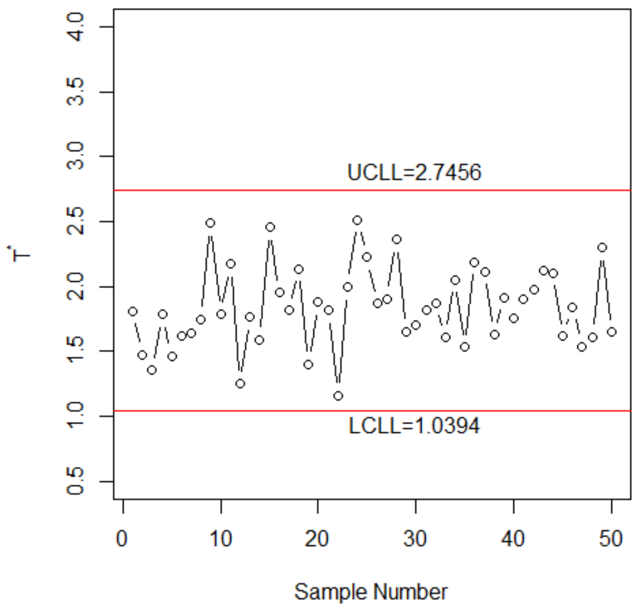

FIGURE 4. The existing control chart for UTIs data.

The values of T_N^* are plotted on the control limit in Figure 3. From Figure 3, it can be read that several values of T_N^* lie in the indeterminacy interval of neutrosophic lower and upper control limits. We also note that sample 11^{th} and 21^{st} are close to LCL_L which needs the hospital management attention. On the other hand, the control chart in Figure 4 under the classical statistics does not show any point indeterminacy interval.

VI. CONCLUDING REMARKS

We presented a control chart when the quality of interest follows the NGD. The proposed control chart is the generalization of the control chart under the classical statistics. The proposed control chart can be applied when the data or the parameters are indeterminate values rather than the exact or determined values. The efficiency of the proposed control chart is discussed using the simulation and the real example. By compression of the proposed chart with the existing control chart, we conclude that the proposed control chart performs better under the uncertainty environment. We recommend that the industrial engineers and health personals should apply the proposed control chart for monitoring the data obtained from the sample or the population having incomplete information. The proposed control chart for the marine big data and ocean big data can be considered as future research.

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